GAME THEORETICAL MODELING FOR PLANNING AND DECISION MAKING

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Military operations are a consistent set of strategic, operational, and tactical actions. Each military conflict is an interaction of hostile parties, which can perform different actions to achieve their goals. In many cases the military conflict consists of offensive actions of one side and defensive actions of the other side. Recently, asymmetric conflicts create many challenges due to their untraditional methods and actions. In all cases the objective of the defense is to minimize the losses caused by the enemy and the objective of the attack is to maximize these losses.

Contemporary strategy and doctrine are based on joint and coalition operations. Operational war-games typically consist of multi-echelon participants as main forces, enemy, control staff, and a number of neutral, friendly and coalition teams. Recently, the Operations Other Than War – peacemaking, peacekeeping, humanitarian relief operations – are of special interest. The asymmetric environment they represent can be modeled in a natural manner using game theory. However, they pose many challenges to the applied game theory in terms of analysis and prediction.

The central part of the model of planning and decision-making is based on the above-mentioned ideas from game theory. Game theory has been chosen due to the fact that it addresses one of the central elements of the process, namely the analysis of alternative courses of action. Planners from each side of the conflict have a separate (and generally different) payoff matrix, representing each planner's perception of the possible courses of action open to him and his opponent, and the consequences of the interaction between them.

The essence of the deliberate planning model[1] is the analysis by the planner of this payoff matrix and the selection of a single course of action that is, in some sense, the 'best' one to take given the perceived options open to the enemy. Selection of a course of action is the command decision and it is the key output of the deliberate planning process model.

Applying modeling software, for example the LINGO-language,[2] gives the opportunity to generate many experiments and to obtain different results. This is very useful in gaining experience through simulation based on historical data. This paper presents practical and fast application of the game theoretic approaches, applied to contemporary asymmetric conflicts.

Game-Theoretical Models

Game theory models provide appropriate mathematical models of real conflict situations.[3] Game theory enables modeling of the most important elements of the planning and decision-making processes – analysis of alternative courses of action, the behavior of the sides, and payoffs and losses. These techniques assist in the optimal allocation of forces and equipment, as well as in making key decisions in operational planning.

Particularly interesting is the game theoretical model of the offensive action. Model development is usually based on different approaches; it depends on the assumed constraints and initial conditions, and it leads to finite or infinite antagonistic game, general positioning game or coalition/non-coalition game, respectively.

Finite Antagonistic Game

A real conflict can be modeled by *finite antagonistic game* if the following conditions are satisfied:

1.1 The conflict is determined by antagonistic interaction of two parties, each of which disposes only finite number of possible actions.

1.2 The parties undertake the actions separately, i.e. each of them does not have information about the operation of the other party. The result of these actions is estimated by a real number that determines the usefulness of the situation for one of the parties.

1.3 Each party evaluates for itself and for the opponent the usefulness of any possible situation, which can develop as a result of their interaction.

1.4 The actions of the parties do not posses formal features. Thus the parties' actions can be treated as abstract homogeneous sets.

If conditions (1.1-1.4) are fulfilled for a given conflict, defining one of the parties by player *I* and the other by player *II*, we can describe the conflict by the following antagonistic game[4]

$$\Gamma = \langle X, Y, H \rangle, \qquad (1)$$

where *X* is the set of pure strategies of player *I*, $X = \{X_1, X_2 \dots X_m\}$;

Y is the set of pure strategies of player *II*, $Y = \{Y_1, Y_2 \dots Y_n\}$;

H is the function of usefulness (payoff) of player *I*, which is determined for all pairs of possible actions of the players.

Real conflicts that satisfy conditions (1.1-1.4) can then be modeled as finite antagonistic game and represented by the following matrix:

$$H = \left\| h_{ij} \right\|, \qquad h_{ij} = H(i,j), \qquad 1 \le i \le m, 1 \le j \le n; \tag{2}$$

In order to find a stable optimal strategy it is necessary to solve the following equations:

$$E_1(X, y_j) = \sum_{i=1}^{n} h_{ij} x_i = const; (j = 1, \dots m); \tag{3}$$

$$E_2(x_i, Y) = \sum_{j=1}^{m} h_{ij} y_j = const; (i = 1, \dots n);$$

$$\sum_{i=1}^{n} x_i = 1; \tag{4}$$

$$\sum_{j=1}^{m} y_j = 1;$$

Thus, the game payoff is:

$$E(X, Y) = \sum_{j=1}^{m} \sum_{i=1}^{n} h_{ij} x_i y_j \,. \tag{5}$$

Strategies $X^* \in X$ and $Y^* \in Y$ are optimal mixed strategies for players *I* and *II*, if the following expression is true:

$E(X, Y^*) \le E(X^*, Y^*) \le E(X^*, Y)$ - expressed as a Cartesian product of the (X, Y) pair.

The solution then is of the following form:

$$\begin{aligned} &\left\| X^*, Y^*, v \right\| \\ &v = E(X^*, Y^*) \end{aligned} \tag{6}$$

where v is the game cost.

Let us presume the following real situation. Side *A* plans an attack against side *B* during time *T*, *T* = *n*t*. The attack begins at moment *i* and goes on at moments *i+1*, *i+2, ..., n.* Simultaneously, side *B* plans to deploy equipment for electronic warfare (EW) and the beginning of this counteraction is the *j*-th time unit and it continues at *j+1*, *j+2, ..., n.*

We presume that if the EW equipment of side *B*, deployed before the attack of side *A*, is disclosed by the intelligence of side *A;* therefore, this equipment becomes ineffective. On the other hand, when the attack of side *A* begins before the deployment of the EW equipment, side A’s weapons’ effectiveness decreases due to this deployment. Thus, the assumption is that the time of the attack and the time of the EW deployment are determined and the attack’s intensity is constant.

Let the expected value (EV) of the number of destroyed ships of side *B* is *EV* = *c*. The assumption is that side *B* does not counterattack side *A*. It is also presumed that if the attack of side A happens simultaneously with the EW usage of side B then *EV = c/2* during the whole time interval *T*. If $i < j$ and side *B* does not counterattack during the time (*j-i),* after that EW will decrease the attack effectiveness to zero.

The EV of the number of destroyed ships at time *T* is *EV* = *c (j-i).* If $i > j$, i.e. side *A* attacks after the EW-deployment, then the EW effectiveness is close to zero and *EV* = *c (n-I+1).* If the actions of both sides happen at the same time (*i=j*), then *EV* = *c (n-i+1)/2*. Side *A* begins the attack at the *i*-th time moment and tries to maximize the number of destroyed ships during the time interval *T*. Side *B* counteracts through their EW equipment at time *j* and tries to minimize the losses.

The described conflict situation is interpreted as offensive action in view of the fact that side *B* could in principle use a military unit, weapons, equipment or maneuvers that can decrease the adversary’s effectiveness.

The relevant mathematical model for this situation is a finite antagonistic game $\Gamma = <x, y, H>$, where *x* = *1, 2, ..., n* is the set of pure strategies of side *A* (player *I*) and *y* = *1, 2, ..., n* is the set of pure strategies of side *B* (player *II*) and the payoff function of player *I* is *H.*

Therefore, according to the above:

$$H = \begin{cases} c^*(j-i) & \text{for } i<j; \\ c^*(n-i+1)/2 & \text{for } i=j; \\ c^*(n-i+1) & \text{for } i>j \end{cases} \tag{7}$$

The related game matrix is the following:

$$H = \begin{pmatrix} n/2 & 1 & 2 \ldots & n-2 & n-1 \\ n-1 & (n-1)/2 & 1 \ldots & n-3 & n-2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & & \\ 2 & 2 & 2 \ldots & 1 & 1 \\ 1 & 1 & 1 \ldots & 1 & 1/2 \end{pmatrix} \tag{8}$$

The solution (X^*, Y^*, v) satisfies the equations:[5]

$$H(X, j) = v, \quad j = 2, 3, 4, \ldots, k+1;$$

$$\xi_1 + \sum_{1}^{K+1} \xi_i = 1; \tag{9}$$

$$H(i, Y) = v, \quad i = 1, 3, 4, \ldots, k+1;$$

$$\sum_{2}^{K+1} \eta_i = 1;$$

When $n=6$ the matrix H and the software model look like:

$$H = \begin{pmatrix} 6/2 & 1 & 2 & 3 & 4 & 5 \\ 5 & 5/2 & 1 & 2 & 3 & 4 \\ 4 & 4 & 4/2 & 1 & 2 & 3 \\ 3 & 3 & 3 & 3/2 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2/2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1/2 \end{pmatrix}$$

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MODEL:
MAX= P;
        A1+A2+A3+A4+A5+A6  = 1;
-P + 3*A1+  5*A2+ 4*A3+  3*A4+ 2*A5+  1*A6 >= 0;
-P + 1*A1+2.5*A2+ 4*A3+  3*A4+ 2*A5+  1*A6 >= 0;
-P + 2*A1+  1*A2+ 2*A3+  3*A4+ 2*A5+  1*A6 >= 0;
-P + 3*A1+  2*A2+ 1*A3+1.5*A4+ 2*A5+  1*A6 >= 0;
-P + 4*A1+  3*A2+ 2*A3+  1*A4+ 1*A5+  1*A6 >= 0;
-P + 5*A1+  4*A2+ 3*A3+  2*A4+ 1*A5+0.5*A6 >= 0;
END
P 2.181818
A1       0.5454545      A4      0.1818182
A2       0.0000000      A5      0.0000000
A3       0.2727273      A6      0.0000000

MODEL:
MIN = P;
B1+B2+B3+B4+B5+B6  = 1;
-P + 3*B1+  1*B2+ 2*B3+  3*B4+ 4*B5+  5*B6 <= 0;
-P + 5*B1+2.5*B2+ 1*B3+  2*B4+ 3*B5+  4*B6 <= 0;
-P + 4*B1+  4*B2+ 2*B3+  1*B4+ 2*B5+  3*B6 <= 0;
-P + 3*B1+  3*B2+ 3*B3+1.5*B4+ 1*B5+  2*B6 <= 0;
-P + 2*B1+  2*B2+ 2*B3+  2*B4+ 1*B5+  1*B6 <= 0;
-P + 1*B1+  1*B2+ 1*B3+  1*B4+ 1*B5+0.5*B6 <= 0;
END
P        2.181818
B1       0.0000000               B4       0.5454545
B2       0.3636364               B5       0.0000000
B3       0.9090909E-01           B6       0.0000000
```

The task is solved by the LINGO-solver and the solution is:

$$X^* = (0.54, 0, 0.27, 0.18, 0, 0), \quad Y^* = (0, 0.36, 0, 0.54, 0, 0), \quad v = 2.18.$$

We can provide the following interpretation. The relevant strategies are: side *A* attacks with a probability 0.54 at the first time unit and with probability 0.27 at the third time units, respectively. Side *B* deploys the EW equipment with a probability 0.36 at the second time unit.

Infinite Antagonistic Game

A real conflict situation can be modeled by *infinite antagonistic game* in case of the following conditions:

2.1 The conflict is determined by antagonistic interaction of two parties where at least one of the parties can initiate infinite number of probable actions.

2.2 The parties initiate the actions in isolation, i.e. they have no information about the operation of the other party. The result of these actions is assessed by a real number, which determines the usefulness of the situation for each of the parties.

2.3 Each party knows the usefulness of any possible situation both for itself and the opponent, which can develop as a result of their interaction.

2.4 The actions of the parties do not posses formal features. Thus, they can be treated as elements of abstract homogeneous sets, which could be distinguished according to the payoff of the game situation.

If the conflict corresponds to (2.1-2.4), defining one of the parties by player *I* and the other by player *II*, we can describe it by the infinite antagonistic game $\Gamma = <X, Y, H>$, where *X* is the set of pure strategies of player *I*, *Y* is the set of pure strategies of player *II*, *H* is the function of usefulness of player *I*, which is determined for all pairs of possible actions of the players.

Continuous game theoretical model that is analogous to the offensive action is the following game. We denote as *t* the beginning of side *A*'s attack against the aircraft-carrier unit of side *B* and as *r* – the moment of side *B*'s actions, namely EW, and $0 \leq t, r \leq T$. Note that $x = t/T$ and $y = r/T$. Then the pure strategy of *A* will be $x \in [0,1]$ and the pure strategy of *B* will be $y \in [0,1]$. The chosen strategies define the game situation (x, y) and the party *A* has the payoff $H(x, y)$.The set of (x, y) situations defines the area $[0, 1]\text{x}[0, 1]$ and the payoff function of player *A* in this area is presented as the following function:

$$H(x,y) = \begin{cases} c*(y-x) & \text{if } x<y; \\ c*(1-x)/2 & \text{if } x=y; \\ c*(1-x) & \text{if } x>y. \end{cases} \qquad (9)$$

We presume that the other conditions are the same as those of the finite antagonistic game. If c = 1 and if we apply some transformations on matrix (7) the result is the following matrix:

$$\left\| H\left(\frac{i-1}{n}, \frac{j-1}{n}\right) \right\|, \qquad 1 \le i, j \le n \qquad (10)$$

where the function H is defined as shown in equation (9).

Thus, the matrix game (7) approximates the infinite game (9), i.e. the conflicts which we model are distinguished only by the nature of the time – in the first case the time is a discrete quantity while in the second case time is a continuous quantity.

The solution of the game described by matrix (10)[6] is $v = 1/e$; the optimal strategy of player *II* on a segment $[0, 1-1/e]$ is defined by the density $1/(1-y)$; the corresponding optimal strategy of player *I* is the cumulative distribution function defined by the density $1/(e(1-x))^2$ on a segment $(0, 1-1/e)$.

General Positioning Game

In principle, real conflicts develop in time and space. Thus, conditions 1.1-1.3 are valid and, moreover, the participants at each phase of the conflict can gather additional information about the situation or, on the contrary, can lose it. The result of their operations can be assessed by a real number, which determines the degree of usefulness of the usual situation for one of the parties. That kind of conflict is modeled by a multi-stage (positioning) game.

Characteristic feature of the application of the positioning games is the construction of a positioning structure of the game and normalization with the subsequent solution in the mixed strategies or strategies of behavior. This feature frequently hampers the application of the game-theoretical methods; to overcome the combinatorial complexity other mathematical means are needed. However, if the number of alternatives is not very large, i.e. the game tree is practically visible, the game, being rather adequate model of dynamics of conflict, allows to obtain nontrivial analysis of the accepted solutions.

It happens in a military conflict one of the sides to have no information about the effectiveness of the other side's weapons. In this case we can consider two aspects of the payoff function: we have a hypothesis of the function or the game theoretical model is developed including the unknown information as a parameter of the strategy. The question is whether this method allows constructing an adequate model of the conflict. One of the approaches leads to the special class positioning antagonistic games with two players. Let we presume the following situation. The players *I* and *II* are opponents with antagonistic interests and can implement finite number of possible actions. The payoff function of player *I* is the set of matrices $H = \{H^1, H^2, \dots H^r\}$. Presume that the first step is a random event and $k \in K = (1,2 \dots m)$. The number k is only known by player *I*. Player *I* also knows the matrix $H^k = \left\| h_{ij}{}^k \right\|$, and chooses one number i, $(i=1,2, \dots m)$. Player *II* knows the set $K = (1, 2, \dots, r)$ and the distribution P_k, $(k \in K)$ and chooses the number j, $(j=1,2 \dots n)$. Having this information player *I* can change his strategy to increase his payoff.

Variations in the choice of players' strategies are based on the available information. Thus, the game is a positioning game with incomplete information – the first step is a random event, player *I* makes the second step and player *II* makes the third (see Figure 1).

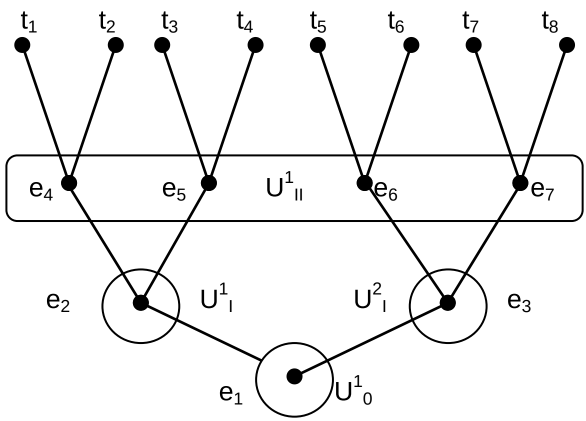

Figure 2: General Positioning Game

The strategy of player I is a function on the family of information sets:

$U_I = \{U^1_I, U^2_I, \dots U^r_I\}$, with values in the interval ($1,m$). I.e. the player's strategy is a set

$(i_1,i_2, \dots i_r)$, where $i_k \in [1,m]$, $k = 1,2,\dots r$ and we can denote the pure strategy as $X(i_1 \dots i_r)$ and

$Card(X) = m^r$.

The second player does not have information about the first and the second action and he only chooses the number j – thus, his strategy is:

$$y_j = (\underbrace{0, \dots, 0}_{j-1},1,0,\dots,0);\ j = 1,2,\dots n;\ \ Card(Y) = n.$$

The optimal strategy for player *I* is defined as:

$X^* = \{\xi^*(x_{i1\dots ir})\}$, where $\xi^*(x_{i1\dots ir})$ is the probability of application of the pure strategy $x_{i1\dots ir}$.

The optimal strategy for player *II* will be:

$Y^* = \{\eta^*(y_j)\}$, where $\eta^*(y_j)$ is the probability of application of the pure strategy y_j.

To illustrate a similar task we presume the single throw finite antagonistic game. An antisubmarine aircraft can use two different tools, 1 and 2, to detect the target. The submarine itself can choose a depth A_1 or A_2 according to the available information. We assume as an effectiveness criterion the probability not to discover the submarine.

Let in the case of condition of type *1* the payoff matrix of player *I* is $H^1 = \left\| h^1{}_{ij} \right\|$, where $h^1{}_{ij}$ is the probability not to discover the submarine in condition *1, i*–th depth and *j*–th tool.

Let in the case of condition of type *2* the payoff matrix of player *I* is $H^2 = \left\| h^2{}_{ij} \right\|$, where $h^2{}_{ij}$ is the probability not to discover the submarine in condition *2, i*–th depth and *j*–th tool.

Player *II* has incomplete information about the situation. Then the payoff matrix is $H^k = \left\| h^k{}_{ij} \right\|$, where $h^k{}_{ij}$ is the probability not to discover the submarine in condition of type *k*, *i*–th depth and *j*–th tool. We suppose that $P_1 = 0.3$ and $P_2 = 0.6$ and the payoff matrices are the following.

$$H^1 = \begin{bmatrix} 0.3 & 0.5 \\ 0.2 & 0.3 \end{bmatrix} \qquad H^2 = \begin{bmatrix} 0.3 & 0.4 \\ 0.6 & 0.2 \end{bmatrix}$$

Let we represent this situation by a positioning game and then construct the payoff table *H* as follows:

H	Y_1	y_2
x_{11}	*0.9/3*	*1.3/3*
x_{12}	*1.5/3*	*0.9/3*
x_{21}	*0.8/3*	*1.1/3*
x_{22}	*1.4/3*	*0.7/3*

```
MODEL:
MAX = PG;
B1+B2+B3+B4=1;
- PG + 0.9*B1+1.5*B2+0.8*B3+1.4*B4 >= 0;
- PG + 1.3*B1+0.9*B2+1.1*B3+0.7*B4 >= 0;
END
```

```
PG    1.14
B1    0.6000
B2    0.4000
B3    0.0000
B4    0.0000
```

```
MODEL:
MIN = LP;
B1+B2=1;
-LP + 0.9*B1+1.3*B2<= 0;
-LP + 1.5*B1+0.9*B2<= 0;
-LP + 0.8*B1+1.1*B2<= 0;
-LP + 1.4*B1+0.7*B2<= 0;
END
```

```
LP    1.14
B1    0.4000
B2    0.6000
```

The problem is then solved by the LINGO-solver and the solution is:

$\xi^*(x_{11}) = 0.6;$ $\xi^*(x_{12}) = 0.4;$ $\xi^*(x_{21}) = 0;$ $\xi^*(x_{22}) = 0;$
$\eta^*(y_1) = 0.4;$ $\eta^*(y_2) = 0.6;$ $v = 0.38.$

The strategy of player *I* is defined as $f^* = ((1,0), (0.6, 0.4)).$

The optimal strategies of the players are:

$\underline{X}^*(1) = (1, 0);$ $\underline{X}^*(2) =(0.6, 0.4);$ $Y^* = (0.4, 0.6).$

The interpretation is the following. In condition of type 1 the best solution is to submerge the submarine at depth A_1. In condition of type 2 the solution is as follows – depth A_1 with probability 0.6 and depth A_2 with probability – 0.4.

Thus, in case of incomplete information it is useful to try to find the optimal solution constructing an appropriate positioning game.

Non-Coalition Game

A real conflict situation can be modeled as a *non-coalition game* if the following conditions are met:

4.1 The conflict is determined by non-antagonistic interaction of parties.

4.2 The parties are not permitted to create coalitions.

4.3 The result of their actions is assessed by a real number that determines the usefulness of the situation for each of the parties.

4.4 Each party knows the usefulness of any possible situation both for itself and the opponent.

If a conflict falls in the category described by (4.1-4.4), we can represent it as a non-coalition game of the following form:

$\Gamma = < I, \{x_i\}, i \in I, \{H_i\}, i \in I >,$

where *I* is the set of players, $\{x_i\}$ is the set of pure strategies of player *i*, $\{H_i\}$ is the payoff function of player *i*, in Cartesian product $x = \Pi_{i \in I} x_i$..

Non-coalition games model real conflict situations when two forces are antagonistic opponents and the benefit of one side is equal to the loss of the other. The theoretical form just presented models the following situation.

An aircraft-carrier unit denoted *A* plans an attack against the aircraft-carrier unit *B* at time T, $T = s^*t$. The attack begins at time moment *i* and continues at moments $i+1, i+2, ..., s$. Simultaneously, *B* plans an attacks at moment $j, j+1, j+2, ..., s$. At the same time, units *A* and *B* deploy radio-electronic countermeasures. We presume, that units *A* and *B* have equal combat capability and that the most important characteristic is the expected value of the number of destroyed enemy ships, namely:

C (in case of attack without counteraction);
c/2 (in case of attack with counteraction);
0 (in case of counteraction through EW).

So, if $i<j$, i.e. side *A* attacks at time interval $(j-i)$ without a counteraction and after that the effectiveness of *A*'s actions becomes zero due to the deployment of enemy's EW. If we accept that *B* begins the attack after the deployment of the EW equipment of side *A* and its effectiveness is close to zero, then the EV of the number of destroyed ships at time *T* is as follows:

$a_{ij} = c*(j-i)$ for *B* and $b_{ij} = c*(s-j+1)$ for *A*.

If $i>j$ then $a_{ij} = c*(s-i+1)$ for *B* and $b_{ij} = c*(i-j)$ for *A*.

If $i=j$ then $a_{ij} = b_{ij} =c*(s-i+1)/2$.

Side *A* begins the attack at the *i*-th time moment and deploys the EW equipment. Then its objective is to maximize the EV of its payoff. Side *B* begins the attack at the *j*-th time moment and counteracts through their EW equipment trying to minimize the losses.

The payoff functions of players *A* and *B*, based on their combat capabilities are as follows:

$$H_A(i,j) = \begin{cases} c*(j-i) & \text{for } i<j; \\ c*(s-i+1)/2 & \text{for } i=j; \\ c*(s-i+1) & \text{for } i>j \end{cases} \quad (11)$$

$$HB(i,j) = \begin{cases} c*(s-j+1) & \text{for } i<j; \\ c*(s-j+1)/2 & \text{for } i=j; \\ c*(i-j) & \text{for } i>j \end{cases} \quad (12)$$

$$HA(i,j) + HB(i,j) = \begin{cases} (s-i+1) & \text{for } i \leq j; \\ (s-j+1) & \text{for } i > j; \end{cases} \quad (13)$$

Thus, according to (13) the players' payoff depends on their strategies. Therefore, the conflict is antagonistic one and is modeled by the antagonistic game $\Gamma = <x, y, H_A, H_B>$, where $x = y = \{1, 2, ..., s\}$ are the sets of pure strategies of the players and H_A, H_B are the payoff functions of the players A and B.

The corresponding matrices are:

$$A = \begin{pmatrix} s/2 & 1 & 2\ldots & s-2 & s-1 \\ s-1 & (s-1)/2 & 1\ldots & s-3 & s-2 \\ c* & & \ldots & & \\ 2 & 2 & 2\ldots & 1 & 1 \\ 1 & 1 & 1\ldots & 1 & 1/2 \end{pmatrix}$$

$$B = c* \begin{pmatrix} s/2 & s-1 & & 2 & 1 \\ 1 & (s-1)/2 & & 2 & 1 \\ \ldots & & & & \\ s-2 & s-3 & \ldots & 1 & 1 \\ s-1 & s-2 & \ldots & 1 & 1/2 \end{pmatrix}$$

The analysis of the bi-matrix game Γ equilibrium state is quite difficult. The task is easier when the players have infinite set of possible strategies. Therefore, let we assume that the time interval T is of the form $[0, 1]$. If player A begins an attack and deploys EW equipment at moment $x \in [0, 1]$ and player B at moment $y \in [0, 1]$, then the players' payoffs are as follows:

$$HA(x,y) = \begin{cases} c*(y-x) & \text{if } x<y; \\ c*(1-x)/2 & \text{if } x=y; \\ c*(1-x) & \text{if } x>y; \end{cases} \qquad (14)$$

$$HB(x,y) = \begin{cases} c*(1-y) & \text{if } x<y; \\ c*(1-y)/2 & \text{if } x=y; \\ c*(x-y) & \text{if } x>y; \end{cases} \qquad (15)$$

where $H_A(x,y) + H_B(x,y) \neq const$

This model is a continuous case analog of the game-theoretical model of the offensive attack – the non-antagonistic infinite game.

Concluding Remarks

There are some particular areas that would benefit from game theory and the other modeling and simulation approaches. The objective often is to assist the planning and the decision-making processes, which are the most important activities in military operations. The perspectives for future development are in the aspects given below.

The analyst needs a suitable tool to automatically enumerate the relevant players, their options, and the estimated payoffs. It is necessary to create and maintain a database, and to combine the expert knowledge. A successful approach is to develop the games from the situation and the historical data. Agent-based modeling could assist these activities with appropriate tools for assessment of the situation, finding of the best alternative, estimation of the payoffs and even planning.

These tools aim at the development of optimal strategies. Similarly, multiplayer game models that reflect effectively the conditions of contemporary conflicts – creating coalitions, international organizations – enlarge the scope of application. Varieties of models correspond to static or dynamic equilibrium. The strategy for improvement is based on the use of expert knowledge of psychological factors. It is important to reuse previous expert assessments of payoffs and previous solutions strategies.

The application of computer-aided software environments (CASE) is a very useful means in the whole process. Modeling languages provide powerful tools to model the conflict situations through the use of game theory. The LINGO language is an automatic tool for optimization and modeling that provides the possibility to solve many discrete and continuous, as well as stochastic tasks. This paper has illustrated the application of the game theoretical models to real situations. Several tasks were solved using the LINGO-software illustrating the usefulness of this commercial-off-the-shelf (COTS) product to military research applications. The strategies are experimented and the solutions are proposed to planners and decision-makers.

Given a game theoretical perspective we begin the process of formulating players, options, and payoffs. Figure 1 illustrates the concept of lifting a hypothetical game from an ABC database[7] of historical events. An ABC database includes selected antecedents to historical events, behaviors or options actually executed by the collected targets, and a valuation of the degree of success or value achieved by the target's action (consequent) for a given set of antecedents and behaviors.

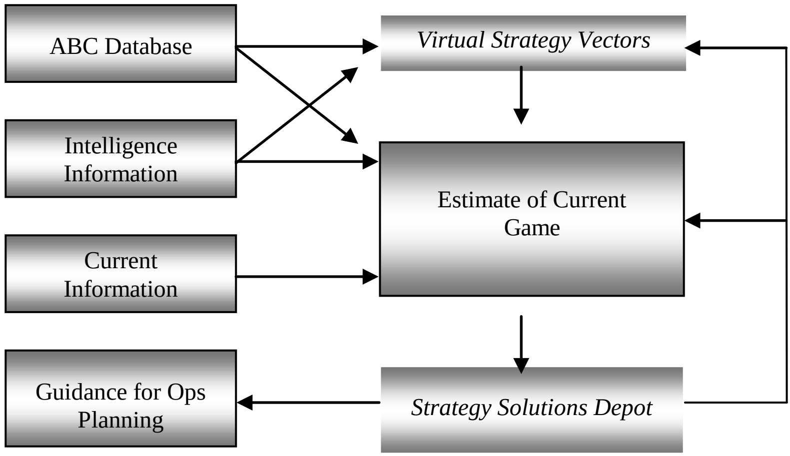

Figure 2: Bootstrapping the Hypothetical Game.

Figure 2 is actually a high-level indication of the process outlined in the research area above. A detailed elaboration is beyond the scope here, but the fundamental notion is that mixed strategy vectors are implicit in the ABC history for each target or player. The process outlined here extracts the implicit strategy vectors and incorporates available intelligence on player ideology, worldview, beliefs, knowledge, capabilities and objectives to generate a plausible set of payoffs. The combination of implicit strategy vectors, plausible payoff matrix and individual player information sets, constitute the initial hypothetical game. Refinement of the initial hypothesis could be directed by reduction of uncertainty in payoff and information estimates and options available to players over time.

In conclusion, using a game theoretic formulation for predictive purposes, we have the problem whether suggested war games representation generates emergent collective behavior that resembles realistic military environment. The assumption of complete information is the greatest impediment to the practical application of classic game theory. An asymmetric information game where players have incomplete information on either payoffs or options or both is much more typical of the real world situation. Preliminary results are encouraging.

Notes:

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[2] *LINGO the Modeling Language and Optimizer* (LINDO Systems Inc., 2001).

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[4] G. N. Dubin and V. G. Suzdal, *Introduction to Applied Game Theory*, in Russian, (Moscow: Science, 1981).

[5] Dubin and Suzdal, *Introduction to Applied Game Theory.*

[6] Dubin and Suzdal, *Introduction to Applied Game Theory.*

[7] Gregory M. Whittaker, *Asymmetric Wargaming: Toward a Game Theoretic Perspective* (The MITRE Corporation, October 2000).

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